

Written Exam for the B.Sc. in Economics winter 2014-15

**Microeconomics C**

Final Exam

Date: 2 January 2015

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

**This exam question consists of 3 pages in total**

PLEASE ANSWER ALL QUESTIONS.  
PLEASE EXPLAIN YOUR ANSWERS.

1. **Nash Equilibrium and Subgame-Perfect Nash Equilibrium.**

(a) Find all the pure and mixed strategy Nash Equilibria of the following game.

		Player 2		
		$t_1$	$t_2$	$t_3$
Player 1	$s_1$	2, 0	3, 2	1, 6
	$s_2$	3, 3	2, 1	0, 2

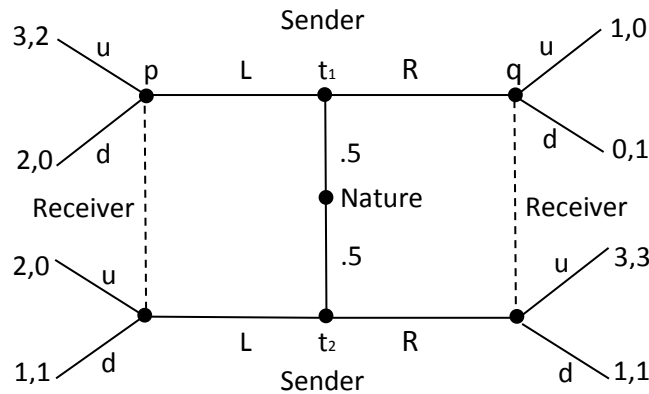
(b) Suppose now that we introduce a new strategy for Player 1. Denote the corresponding game by  $G$ :

		Player 2		
		$t_1$	$t_2$	$t_3$
Player 1	$s_1$	2, 0	3, 2	1, 6
	$s_2$	3, 3	2, 1	0, 2
	$s_3$	1, 4	10, 10	0, 12

Use iterated elimination of strictly dominated strategies to simplify the game. Explain briefly each step (1 sentence). What is the set of Nash Equilibria of  $G$ ? (*Hint*: No new calculations are required.)

(c) Now suppose we repeat  $G$  twice. Denote the resulting game by  $G(2)$ . How many proper subgames are there (not counting the game itself)? Show that there is a Subgame-perfect Nash Equilibrium of  $G(2)$  in which  $(s_3, t_2)$  is played in stage 1.

2. **Signaling.** Consider the following signaling game.



- (a) Find all the (pure strategy) separating Perfect Bayesian Equilibria (PBE).
- (b) Find the (pure strategy) pooling PBE in which both types send message  $L$ . Does it satisfy signaling requirement 5 (SR5)? Explain briefly.
- (c) Suppose you are a politician and you want to prove that you are trustworthy and incorruptible.
  - i. Give an example of a signal that is not credible and explain briefly (1 sentence) why it is not credible.

- ii. Give an example of a signal that is credible and explain briefly (1 sentence) why it is credible.

3. **Coalitions.** Three entrepreneurs are considering starting a new tech company. They are free to form a company of any size between themselves. Entrepreneurs A and B are very experienced, with A being slightly more experienced than B, whereas entrepreneur C has no experience whatsoever. If entrepreneurs A and B work together in the company, the value of the company is 5000 gazillion dollars (regardless of whether entrepreneur C joins the company). If entrepreneur A starts the company alone or with C, it is worth 2000 gazillion dollars. If entrepreneur B starts the company alone or with C, it is worth 1000 gazillion dollars. If entrepreneur C starts the company alone, it is worth 0 gazillion dollars.

- (a) Think of this situation as a coalitional game with transferable payoffs. Write down the value of each coalition.
- (b) Find the core of this game.

4. **Spence education model.** Consider the following version of Spence's education signaling model, where a firm is hiring a worker. Workers are characterized by their type  $\theta$ , which measures their ability. There are two worker types:  $\theta \in \{\theta_L, \theta_H\}$ . Nature chooses the worker's type, with  $p_H = \mathbb{P}(\theta = \theta_H)$  and  $p_L = \mathbb{P}(\theta = \theta_L) = 1 - p_H$ . The worker observes his own type, but the firm does not.

The worker can choose his level of education:  $e \in \mathbb{R}^+$ . The cost to him of acquiring this education is

$$c_\theta(e) = 2 \cdot \frac{e}{\theta}.$$

Education is observed by the firm, who then forms beliefs about the worker's type:  $\mu(\theta|e)$ . We assume that the marginal productivity of a worker is equal to his ability and that the firm is in competition such that it pays the marginal productivity:  $w(e) = \mathbb{E}(\theta|e)$ . Thus, the payoff to a worker conditional on his type and education is

$$u_\theta(e) = w(e) - c_\theta(e).$$

Suppose for this exercise that  $\theta_H = 3$  and  $\theta_L = 1$ .

- (a) In a separating equilibrium the low-ability worker chooses education level  $e_L$  and obtains wage  $w_L = w(e_L)$ . Is it possible that  $e_L > 0$ ? Explain briefly (max. 3 sentences).
- (b) Find a separating pure strategy Perfect Bayesian Equilibrium where the two types choose education levels  $e_L$  and  $e_H$ , respectively, and the low ability type is indifferent between choosing  $e_L$  and  $e_H$ . Assume that off the equilibrium path, the firm assigns zero probability to the worker being type  $\theta_H$ .
- (c) Let  $p = p_H$ . Find a pooling pure strategy Perfect Bayesian Equilibrium in which both types choose education level  $\bar{e}$ , and the low ability type is indifferent between choosing  $e = 0$  and  $e = \bar{e}$ . Assume that off the equilibrium path, the firm assigns zero probability to the worker being type  $\theta_H$ . Does the pooling equilibrium of (c) satisfy SR6? You can show this either graphically or algebraically.